Date

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Social Choice Theory

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Arrows Theorem

The Impossibility Theorem, when $m \geq 3$, there isnt a SWF that satisfy all four

- Unrestricted domain (U)
- Pareto Principle (P)
- Non-dictatorship (ND)
- Independence of Irrelevant Alternatives (IIA)

IIA Senario

Using the plurality rule

Scenario 1

40%	25%	35%
Trump	Rubio	Kasich
Kasich	Kasich	Trump
Rubio	Trump	Rubio

Results: $Trump \succ Kasich$

Scenario 2

40%	25%	35%
Trump	Kasich	Kasich
Kasich	Trump	Trump
Rubio	Rubio	Rubio

Results: $Kasich \succ Trump$

Rubio acts as a siphon/spoiler

IIA Definition

Social preference between two alternatives x and y should depend only on individuals preference between x and y, and not on preferences concerning some third alternative.

Problems

- stronger than need to prevent spoilers
- sensitivity to preference intensities impossible

Anti-Spoiler Rationale vs Sensitivity

Under Borda count where a candidate gets m points for every 1st rank, $m-1$ points for every 2nd rank, etc.

Scenario A

45%	55%
x	y
z	x
y	z

Social Ranking: $x \succ y \succ z$

Scenario B

45%	55%
x	y
y	x
z	z

Social Ranking: $y \succ x \succ z$

Small gap vs Big gap (Sensitivity)

Modified IIA

Given alternatives x and y and two profiles

- each individual ranks x and y the same way in both profiles
- each individual ranks the same set of alternatives between x and y in both profiles

Then the social ranking of x and y must be the same.

May's Theorem

m=2
Majority rule is best.
Satisfy:

- Anonymity (A)
- Neutrality (N)
- Positive Responsiveness (PR)

Borda Count

Alternative x is socially preferred to y if and only if x's Borda score is bigger.

An alternative gets m points every time an individual ranks it first, $m - 1$ points every time an individual ranks it second, etc

Only Borda count - U,P,ND, MIIA,A,N and PR

Proof

When $|X| = 2$, May's theorem = Borda count.

Let $X = \{x, y, z\}$
For profile $\succ_.$
$a_{xy}(\succ_.)$ – $(x \succ y \succ z)$ or $(z \succ x \succ y)$
$a_{yx}(\succ_.)$ – $(z \succ y \succ x)$ or $(y \succ x \succ z)$

For $I_3^F(a_{xy}, a_{yx})$ be proportion a_{xzy}
If $a_{yzx}(\succ_.) = 1 - a_{xy} - a_{yx} - I_3^F(a_{xy}, a_{yx})$, then $x \sim_F y$, where $\succeq_F = F(\succ_.)$
I_3^FSocial Indifference Curve

Continue Proof

Fractions does not affect the social ranking of x and y

$a_{xy}(\succ_{.})$ – $(x \succ y \succ z)$ or $(z \succ x \succ y)$

Actual division does not matter

$I_3^F(a_{xy}, a_{yx})$ is unique.

Two values: a_{xzy} and $a^{'}_{xzy}$ with $a_{xzy} < a^{'}_{xzy}$

From $(a_{xy}, a_{yz}, a_{xzy}, 1 - a_{xy} - a_{yz} - a_{xzy})$ to $(a_{xy}, a_{yz}, a'_{xzy}, 1 - a_{xy} - a_{yz} - a'_{xzy})$ means x is rising and y is falling in individual rankings.

$x \sim_F y$ for both is not possible.

CONTINUE PROOF

$$I_3^B(a_{xy}, a_{yx}) = (2 - 3a_{xy} - a_{yx})/4 \quad (1)$$

$$I_3^F = I_3^B \quad (2)$$

$$I_3^F(a_{xy}, a_{yx}) = B_0 + B_{xy}a_{xy} + B_{yx}a_{yx}, \text{ for constant } B_0, B_{xy}, B_{yx} \quad (3)$$

$$I_3^F(a, a) = 1 - 2a - I_3^F(a, a) \text{ if } a_{xy} = a_{yx} = a \quad (4)$$

$$2I_3^F(a, a) = 2B_0 + 2(B_{xy} + B_{yx})a = 1 - 2a(3) \text{ into } (4) \quad (5)$$

Continue Proof

For a under a certain size, (5) holds and we can infer

$$B_0 = \frac{1}{2} \tag{6}$$

$$B_{xy} + B_{yx} = -1 \tag{7}$$