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Social Choice Theory

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Lemma 3.5

Assume that for all integers $n \geq 2$ and for every n-ary aggregator $\bar{f} = (f_1, ..., f_m)$, there is an integer $d \leq n$ such that for every integer $j \leq m$ and every two-element subset $B_j \subseteq X_j$. the restriction $f_j|B_j$ is equal to pr_d^n, the n-ary projection on the d-th coordinate.

Then for all integers $n \geq 2$ and for every n-aary aggregator $\bar{f} = (f_1, ..., f_m)$ and for all $s \geq 2$, there is an integer $d \leq n$ such that for every integer $j \leq m$ and every subset $B_j \subseteq X_j$ of cardinality at most s, the restriction $f_j|B_j$ is equal to pr_d^n.

N=4,S=3,M=2

	m1	m2
n1	a	a
n2	b	b
n3	c	c
n4	a	a

Let $f_1(a, b, c, a) = b$ and $f_2(a, b, c, a) = c$.
If among x_1, x_2, x_3, x_4 at most two are different then $f(x_1, x_2, x_3, x_4) = x_1$

N=4,S=3,M=2- CONTINUE

Set $g(x_1, x_2) = f(x_1, f(x_1, x_2, c, x_1), c, x_1)$

$$g_1(a, b) = f_1(a, f_1(a, b, c, a), c, a)$$
$$g_1(a, b) = f_1(a, b, c, a)$$
$$g_1(a, b) = b$$

$$g_2(a, b) = f_2(a, f_2(a, b, c, a), c, a)$$
$$g_2(a, b) = f_2(a, c, c, a)$$
$$g_2(a, b) = a$$

N=5,S=3,M=2

	m1	m2
n1	a	a
n2	b	b
n3	c	c
n4	a	a
n5	a	a

Let $f_1(a, b, c, a, a) = b$ and $f_2(a, b, c, a, a) = c$.

If among x_1, x_2, x_3, x_4, x_5 at most two are different then $f(x_1, x_2, x_3, x_4, x_5) = x_1$

N=5,S=3,M=2 CONTINUE

Set $g(x_1, x_2) = f(x_1, f(x_1, x_2, c, x_1, x_1), c, x_1, x_1)$

$$g_1(a, b) = f_1(a, f_1(a, b, c, a, a), c, a, a)$$
$$g_1(a, b) = f_1(a, b, c, a, a)$$
$$g_1(a, b) = b$$

$$g_2(a, b) = f_2(a, f_2(a, b, c, a, a), c, a, a)$$
$$g_2(a, b) = f_2(a, c, c, a, a)$$
$$g_2(a, b) = a$$

N=5,S=4,M=2

	m1	m2
n1	a	a
n2	b	b
n3	c	c
n4	d	d
n5	a	a

Let $f_1(a, b, c, d, a) = b$ and $f_2(a, b, c, d, a) = c$.

If among x_1, x_2, x_3, x_4, x_5 at most three are different then $f(x_1, x_2, x_3, x_4, x_5) = x_1$

N=5,S=4,M=2 CONTINUE

Set $g(x_1, x_2, x_3) = f(x_1, f(x_1, x_2, x_3, d, x_1), x_3, d, x_1)$

$g_1(a, b, c) = f_1(a, f_1(a, b, c, d, a), c, d, a)$
$g_1(a, b, c) = f_1(a, b, c, d, a)$
$g_1(a, b, c) = b$

$g_2(a, b, c) = f_2(a, f_2(a, b, c, d, a), c, d, a)$
$g_2(a, b, c) = f_2(a, c, c, d, a)$
$g_2(a, b, c) = a$

N=5,S=4,M=3

	m1	m2	m3
n1	a	a	a
n2	b	b	b
n3	c	c	c
n4	d	d	d
n5	a	a	a

Let $f_1(a, b, c, d, a) = b$, $f_2(a, b, c, d, a) = c$ and $f_3(a, b, c, d, a) = a$.

If among x_1, x_2, x_3, x_4, x_5 at most three are different then $f(x_1, x_2, x_3, x_4, x_5) = x_1$

N=5,S=4,M=3 CONTINUE

Set $g(x_1, x_2, x_3) = f(x_1, f(x_1, x_2, x_3, d, x_1), x_3, d, x_1)$

$$g_1(a, b, c) = f_1(a, f_1(a, b, c, d, a), c, d, a)$$
$$g_1(a, b, c) = f_1(a, b, c, d, a)$$
$$g_1(a, b, c) = b$$

$$g_2(a, b, c) = f_2(a, f_2(a, b, c, d, a), c, d, a)$$
$$g_2(a, b, c) = f_2(a, c, c, d, a)$$
$$g_2(a, b, c) = a$$

$$g_3(a, b, c) = f_3(a, f_3(a, b, c, d, a), c, d, a)$$
$$g_3(a, b, c) = f_3(a, a, c, d, a)$$
$$g_3(a, b, c) = a$$