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Social Choice Theory

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Lemma 3.5

Assume that for all integers $n \geq 2$ and for every n-ary aggregator $\bar{f} = (f_1, ..., f_m)$, there is an integer $d \leq n$ such that for every integer $j \leq m$ and every two-element subset $B_j \subseteq X_j$. the restriction $f_j|B_j$ is equal to pr_d^n, the n-ary projection on the d-th coordinate.

Then for all integers $n \geq 2$ and for every n-aary aggregator $\bar{f} = (f_1, ..., f_m)$ and for all $s \geq 2$, there is an integer $d \leq n$ such that for every integer $j \leq m$ and every subset $B_j \subseteq X_j$ of cardinality at most s, the restriction $f_j|B_j$ is equal to pr_d^n.

Semi-Projection Function

Let A be a set and let $f : A^3 \mapsto A$ be a function such that if among x_1, x_2, x_3 at most two are different then $f(x_1, x_2, x_3) = x_1$. Assume that there exist pairwise distinct a_1, a_2, a_3 such that $f(a_1, a_2, a_3) = a_2$

Define $g(x_1, x_2) = f(x_1, f(x_1, x_2, a_3), a_3)$

Proof of Supportive

$g(x_1, x_2) = f(x_1, f(x_1, x_2, a_3), a_3)$

Case 1: $f(x_1, x_2, a_3) = x_1$
Case 2: $f(x_1, x_2, a_3) = x_2$
Case 3: $f(x_1, x_2, a_3) = a_3$

Continue
Case 1: $f(x_1, x_1, a_3) = x_1 \in \{x_1, x_2\}$
Case 2: $f(x_1, x_2, a_3) = x_2 \in \{x_1, x_2\}$
Case 3: $f(x_1, a_3, a_3) = x_1 \in \{x_1, x_2\}$

Proof of Semi-Projection

$g(x_1, x_2) = f(x_1, f(x_1, x_2, a_3), a_3)$

Case 1: $g(a_1, a_2) = a_2$

Case 2: $g(a_1, a_3) = a_1$

Proof for Lemma 3.5

Instead of proving $X \Rightarrow Y$ we will prove the negation which is $\neg Y \Rightarrow \neg X$

Proof Continue

$[X]$ Hypothesis: If for every $n \geq 2$,$m = 2$,$s = 2$

$[Y]$ Then: It is true for $n = 3, m = 2, s = 3$

Proof Continue

	m1	m2
n1	a	a
n2	b	b
n3	c	c

Let $f_1(a, b, c) = a$ and $f_2(a, b, c) = b$ giving us $[\neg Y]$

Similarly using the definition $g(x_1, x_2) = f(x_1, f(x_1, x_2, a_3), a_3)$, we get $g_1(a, b) = a$ and $g_2(a, b) = b$ which is the negation of the hypothesis $[\neg X]$

Proof Continue

[X] Hypothesis: If for every $n \geq 2$,$m = 2$,$s = 2$

[Y] Then: It is true for $n = 4, m = 2, s = 3$