Date

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Social Choice Theory

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Motivation and Relation to Aggregation Theory

- m issues
- n voters
- A_j is the possible positions of for issue j
- Domain $X \subseteq \Pi_{j=1}^{m} A_j$

Motivation and Relation to Aggregation Theory

- $\bar{x}$ is an element of $(\Pi_{j=1}^{m} A_j)^n$
- x_j^i
- n-ary aggregator $F : (\Pi_{j=1}^{m} A_j)^n \mapsto \Pi_{j=1}^{m} A_j$
- $n = 2$ or $n = 2$
- Closed Domain: if $\bar{x} \in X^n$, then $F(\bar{x}) \in X$

Assumption

Aggregator satisfy IIA

There exist functions $f_1, ..., f_m$ such that $f_i : A_j^n \mapsto A_j$ and $F(\bar{x}) = (f_1(x_1), ..., f_m(x_m))$

Systematicity: $f_1 = f_2 = ... = f_m$

Definition and Previous Results

- $I = \{1, ..., m\}$
- $\mathcal{A} = \{A_1, ..., A_m\}$ with cardinality at least 2
- Boolean Framework
- $X_j \subseteq \Pi_{j=1}^{m} A_j$

Definitions and Prev Work

- n-ary aggregator $F : (\Pi_{j=1}^{m} A_j)^n \mapsto \Pi_{j=1}^{m} A_j$
- Closed Domain: if $\bar{x} \in X^n$, then $F(\bar{x}) \in X$
- F is dictatorial if $\exists d \in \{1, ..., n\} \ni \forall(\bar{x} \in X^n | F(\bar{x}) = x^d)$
- Possibility Domain: $\exists$ a non-dictatorial aggregator of arity n

Properties of Aggregator

Anonymous: every $\bar{x}, \bar{y} \in X^n$ such that $y^1, ..., y^n$ are a permutation of $x^1, ..., x^n$, then $F(\bar{x}) = F(\bar{y})$

Systematicity: $f_1 = f_2 = ... = f_m$

Independence: There exist functions $f_1, ..., f_m$ such that $f_i : A_j^n \mapsto A_j$ and $F(\bar{x}) = (f_1(x_1), ..., f_m(x_m))$

Properties of Aggregator

Supportive: for every $\bar{x} \in X^n, f_i(x_j) \in \{x_j^1, ..., x_j^n\}$

Paretian: if every $\bar{x} \in X^n$ and every $j = 1, ..., m$ if $x_j^1 = ... = x_j^n$ then $f_j(x_j) = x_j^1 = ... = x_j^n$

Ternary Operation

$f : A^3 \mapsto A$

Majority Operation

$$f(x, x, y) = f(x, y, x) = f(y, x, x) = x$$

Minority Operation

$$f(x, x, y) = f(x, y, x) = f(y, x, x) = y$$

Admits Ternary Aggregator: if there is a ternary aggregator $\bar{f} = (f_1, ..., f_m)$ for all X

Dealing with 2 element subset

$$maj(x, y, z) = \begin{cases} x & \text{if x=y or x=z} \\ y & \text{if y=z} \end{cases}$$

$$min(x, y, z) = \begin{cases} z & \text{if x=y} \\ x & \text{if y=z} \\ y & \text{if x=z} \end{cases}$$

Unsure What This Mean

X admits a minority aggregator if and only if X is an affine logical relations, a subset of $\{0, 1\}^m$ that is the set of solutions of linear equations over the two-element field.

Example 2.4

Set $X = \{(a,a,a),(b,b,b),(c,c,c),(a,b,b),(b,a,a),(a,a,c),(c,c,a)\}$ admits a majority aggregator.

$$maj(u,v,w) = \begin{cases} a & \text{if u,v,w are pairwise different} \\ maj(u,v,w) & \text{otherwise} \end{cases}$$

$x = (a,b,b)$ $y = (a,a,c)$ $z = (c,c,a)$
Then $\bar{f}(x,y,z) = (f(a,a,c), f(b,a,c), f(b,c,a)) = (a,a,a) \in X$

Example 2.5

Set $X = \{(a,b,c),(b,a,a),(c,a,a)\}$ admits minority aggregator

$$min(u,v,w) = \begin{cases} a & \text{if u,v,w are pairwise different} \\ min(u,v,w) & \text{otherwise} \end{cases}$$

$x = (a,b,c)$ $y = (b,a,a)$ $z = (c,a,a)$
Then $\bar{f}(x,y,z) = (f(a,b,c), f(b,a,a), f(c,a,a)) = (a,b,c) \in X$

Example 2.6

Set $W = \{(1,0,0),(0,1,0),(0,0,1)\}$ is an impossibility domain.

Check using majority and minority aggregator..

Conditions to be a possibility Domain

- X is a possibility domain
- X admits a non-dictatorial ternary aggregator