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Social Choice Theory

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PSR

Positional Voting Rules - Ranked Voting Electoral System

Use Ranked Ballot

- anonymous
- value of first preference > value of last preference
- value of n^{th} preference $\geq$ value of $n+1^{\text{th}}$ preference

Example of PSR

Harmonic Progression: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots$

Nauru System: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

How Borda Fit

Points for position in rank(n)

Weights: $w_n = a - (n - 1)$ where a=N

Rank	Points
1	5
2	4
3	3
4	2
5	1

Example of PSR That Don't Work with MIIA

Plurality Voting: The most preferred option receives 1 point; all other options receive 0 points.

Rank	Points
A	1
B	0
C	0

Profile 1 B$\sim C$

Rank	Points
B	1
C	0
A	0

Profile 2 B$\succ C$

Example 2 of PSR That Don't Work with MIIA

Anti-Plurality Voting: Least preferred receives 0 points, everyone else receives 1 point.

Rank	Points
A	1
B	1
C	0

Profile 1 B$\succ C$

Rank	Points
B	1
C	1
A	0

Profile 2 B$\sim C$

PSR with no equivalence

- anonymous
- value of first preference > value of last preference
- ~~value of n^{th} preference $\geq$ value of $n+1^{\text{th}}$ preference~~
- value of n^{th} preference > value of $n+1^{\text{th}}$ preference

Rule of Half

The first preference receives a points, the second receives $\frac{a}{2}$, and so on.

Points	1 Voter	1 Voter
8	A	C
4	B	B
2	C	A

Profile 1 C$\succ B$

Points	1 Voter	1 Voter
8	A	A
4	B	C
2	C	B

Profile 2 C$\sim B$

Only Borda count satisfy Unrestricted Domain (U), Anonymity (A), Neutrality(N), Positive Responsiveness (PR) and Modified Independence of Irrelevant Alternatives (MIIA)

PSR and A,U and N

Trivial that PST satisfy U, A and N.

PSR and PR

Positive Responsiveness requires that if alternative x rises relative to y in some individuals' preference ordering, then

- x doesn't fall relative to y in the social orderings
- if x and y were previously tied socially, x is now strictly above.

Not Every PSR satisfy PR

Points	1 Voter	1 Voter
1	A	A
0	B	B
0	C	C

Profile 1 B$\sim C$

Points	1 Voter	1 Voter
1	A	A
0	B	C
0	C	B

Profile 2 B$\sim C$

Assume a PSR where given two adjacent rank position, the lower one is receives strictly lower amount of points than the higher one.

Value per n^{th} rank: $p - x_n$ where $x_i < x_{i+1}$ for all $i > 1$

When alternative x rises relative to y: $p - x_j$ to $p - x_k$ where $(p - x_j) < (p - x_k)$

No more ties if was tied before.

Example PSR that satisfy PR

- Borda Count $n, n-1, n-2, ..., 1$
- Nauru Method $1, \frac{1}{1-d}, \frac{1}{1-2d}, \dots$
- Harmonic Progression $1, \frac{1}{2}, \frac{1}{3}....$

New MIIA

Given alternatives x and y and two profiles

- each individual ranks x and y the same way in both profiles
- each individual ranks the **same set of alternatives** between x and y in both profiles

Then the social ranking of x and y must be the same.

Change 2nd statement to general form

- each individual have the same number of alternatives between x and y

Theorem

SWF F satisfies U, A, N, PR and NMIIA if and only if F is the Borda Count.

Arithmetic Progression

Pseudo-Proof/Intuition

The case for when the Borda Count satisfy all 5 criteria is straightforward.

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Let there be a profile 1 with 5 alternatives.
Let everyone vote $a_1 \succ x \succ a_2 \succ a_3 \succ y$.

For profile 2, everyone votes the same except 1 voter who votes $a_2 \succ x \succ a_1 \succ a_3 \succ y$

For profile 3, everyone votes the same as profile 1 except 1 voter who votes $x \succ a_1 \succ a_2 \succ y \succ a_3$

Further Work

How the number of alternatives between x and y can be linked to the Nauru system or other system.